# A Robust Six- to Four- Port Reduction Algorithm

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## ABSTRACT

The Six- to Four- Port Reduction technique has been enhanced in a number of ways. A more stable iteration strategy is presented, which shows improved performance in the presence of noise, and with non - ideal microwave hardware. The quality of the five Six- to Four- Port Reduction coefficients is also quantified by a new Variance technique, which allows their effect on reflection coefficient measurement accuracy to be predicted.

## INTRODUCTION

The Six- to Four- Port Reduction technique proposed by Engen[2] has been widely accepted as an attractive method for the calibration of six - port reflectometers, since it only requires three precision reflection standards. However, the iterative solution of the non-linear equations used in the technique can diverge if the power measurements have greater than 1% noise, and less for a poorly behaved microwave circuit.

This paper documents the performance of the Six- to Four-Port Reduction through software simulation, showing the effect that noise during calibration has on measurement accuracy. A new figure of merit is proposed, which indicates the quality of the five Six- to Four- Port Reduction coefficients, based on the variance of circle intersection errors in the measurements used for the calibration. The variance helps to give an indication of the measurement quality that can be expected after the calibration.

In addition, the variance has also been applied to a new Six- to Four- Port Reduction algorithm, which exhibits an improved stability of convergence in the presence of noise or non-ideal microwave circuitry. This technique is combined with the original Six- to Four- Port Reduction algorithm in two forms, to create a robust strategy with the following properties:

- Higher convergence likelihood in the presence of noise.
- Automatic choice of the best initial estimate for iteration.
- No speed degradation under normal conditions.
- Coefficient quality is quantified after iteration, including iteration divergence.

Details of the new six - to four- port reduction strategy are presented, together with practical results from a 250MHz to 26.5GHz co-axial six - port reflectometer.

## 1. THE CONVENTIONAL SIX- TO FOUR- PORT REDUCTION ALGORITHM

The six- to four-port reduction method was first developed by Engen[1,2] and subsequently by Griffin et al[3]. It consists of partitioning the six-port into a "perfect" four-port reflectometer and a two-port network whose S-parameters characterise the errors in measurement between the four-port and the device under test (DUT). This is shown in figure 1.1.

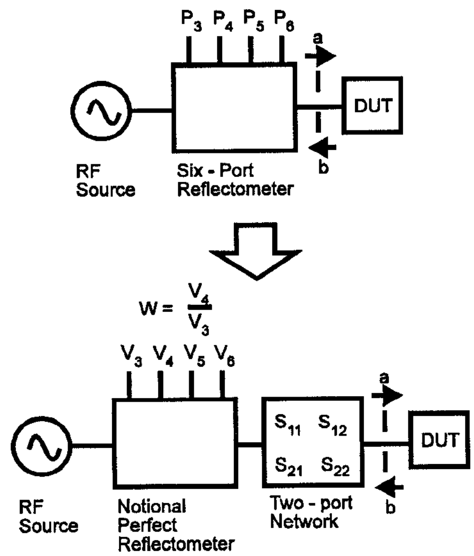


*Figure 1.1 · Partitioning the Six - Port Reflectometer*

The act of partitioning the six- port in this way also partitions the eleven coefficients used to characterise it. Three complex numbers describe the two- port network. The notional perfect reflectometer is defined by finding the five remaining six- port coefficients. The four power readings are combined with the five "reduction" coefficients ($p$, $q$, $r$, $A^2$, $B^2$) to calculate the complex ratio $W$ that the perfect reflectometer would have measured:

$$W = u + jv \tag{1.1}$$

where

$$u = \frac{Q_1 - A^2 Q_2 + r}{2\sqrt{r}} \tag{1.2}$$

$$v = \frac{r(p+q-r)+(p-q+r)Q_1-(p-q-r)A^2Q_2-2rB^2Q_3}{\pm 2\sqrt{r(2pq+2qr+2rp-p^2-q^2-r^2)}} \tag{1.3}$$

and

$$Q_1 = \frac{P_4}{P_3}, \qquad Q_2 = \frac{P_5}{P_3}, \qquad Q_3 = \frac{P_6}{P_3} \tag{1.4}$$

Engen[2] has shown that these five coefficients may be computed after observing the responses to a set of arbitrary and unknown reflection coefficients. This is possible because a constraint equation may be formed between $Q_1$, $Q_2$ and $Q_3$ which does not involve the reflection coefficient of the DUT.

$$\begin{aligned} &pQ_1^2 + qA^2Q_2^2 + rB^2Q_3^2 + \\ &(r-p-q)A^2Q_1Q_2 + (q-p-r)B^2Q_1Q_3 + (p-q-r)A^2B^2Q_2Q_3 + \\ &p(p-q-r)Q_1 + q(q-p-r)A^2Q_2 + r(r-p-q)B^2Q_3 + pqr = 0 \end{aligned} \tag{1.5}$$

Equation (1.5) is used as the basis for the six- to four- port reduction. In practice, nine or more arbitrary loads are used, in order to obtain a closed - form initial estimate of the reduction coefficients. These values are then improved by iterating (1.5) with a least- squares technique[3].

Divergence of the reduction iteration using (1.5) is where $p,q$ and $r \Rightarrow 0$, since that causes the equation to evaluate to zero regardless of the values of $A^2$ and $B^2$, or the $Q_i$. Hence it is a false root of (1.5). The iteration strives to minimise a sum of squared residuals defined by:-

$$S = \sum_{i=1}^{n} \left( \begin{aligned} &pQ_1^2 + qA^2Q_2^2 + rB^2Q_3^2 + \\ &(r-p-q)A^2Q_1Q_2 + (q-p-r)B^2Q_1Q_3 + (p-q-r)A^2B^2Q_2Q_3 + \\ &p(p-q-r)Q_1 + q(q-p-r)A^2Q_2 + r(r-p-q)B^2Q_3 + pqr \end{aligned} \right)^2 \tag{1.6}$$

where $n$ is the number of arbitrary terminations. Note that every term is multiplied by some function of $p$, $q$, and $r$. A modified minimisation function which does not exhibit the same false roots as (1.6) is

$$S = \sum_{i=1}^{n} \frac{\left( \begin{aligned} &pQ_1^2 + qA^2Q_2^2 + rB^2Q_3^2 + \\ &(r-p-q)A^2Q_1Q_2 + (q-p-r)B^2Q_1Q_3 + (p-q-r)A^2B^2Q_2Q_3 + \\ &p(p-q-r)Q_1 + q(q-p-r)A^2Q_2 + r(r-p-q)B^2Q_3 + pqr \end{aligned} \right)^2}{(pqr)^2} \tag{1.7}$$

When $p,q$ and $r \Rightarrow 0$, equation (1.7) becomes large, and hence is more stable than (1.6). There are other false roots near to the correct solution, so this must be used with caution.

## 2. A New Reduction Equation

From a geometrical viewpoint, it can be seen from figure 2.1 that when the correct set of reduction coefficients have been found, the three circles will always intersect in a single point, whatever the value of the termination.

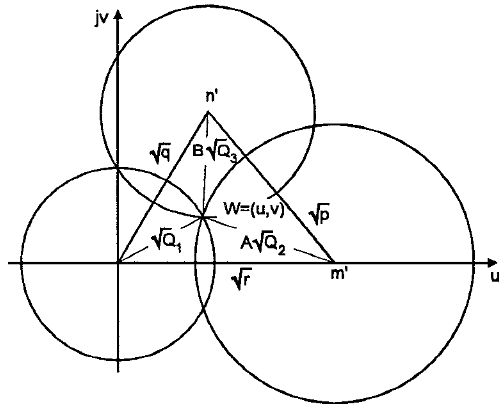


*Figure 2.1 : Six - Port Circle Intersection in W - Plane*

The converse can be used for a new iterative method. Using an expression for the degree of non- intersection of the three circles for each arbitrary termination, a minimisation iteration can be used to find the reduction coefficients. Numerical simulation shows that this technique has a wider capture range for convergence than equation (1.5), which is attributed to the lower powers of the variables involved.

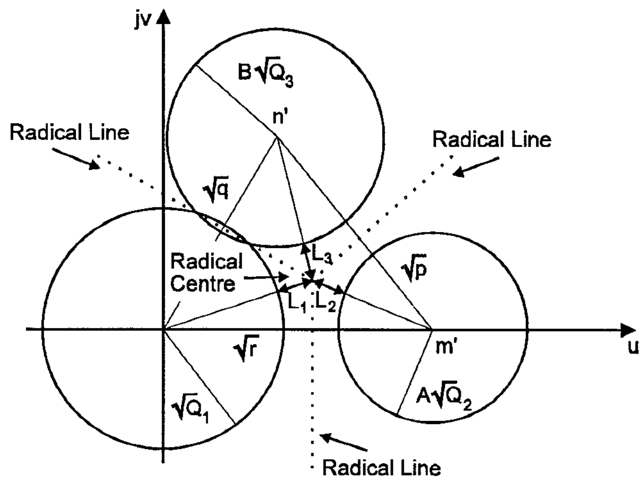


*Figure 2.2 : Six- to Four- Port Reduction using Distance to Radical Centre*

The minimisation function is obtained by calculating the difference between the radical centre and the nearest point on each of the circles. The radical centre is a defined point for any set of three circles, and only touches all three circles when they all intersect in one point. The lengths marked $L_1$, $L_2$, and $L_3$ in figure 2.2 are used to construct the new variance equation:

$$V = \sum_{i=1}^{n} L_{i1}^2 + L_{i2}^2 + L_{i3}^2 \tag{2.1}$$

where $i$ is one of $n$ arbitrary terminations. The co-ordinates of the radical centre can be found by evaluating (1.2) and (1.3) using the current estimates for the five reduction coefficients. The positive root for (1.3) is assumed at this stage, even if the correct root is known to be negative. The sign choice may be deferred until the coefficients have been found. Let the co-ordinates of the radical centre be denoted by $u_{rc}$ and $v_{rc}$. Now $L_1$, $L_2$, and $L_3$ can be found by invoking Pythagoras:-

$$L_1 = \sqrt{u_{rc}^2 + v_{rc}^2} - \sqrt{Q_1} \tag{2.2}$$

$$L_2 = \sqrt{(\sqrt{r} - u_{rc})^2 + v_{rc}^2} - A\sqrt{Q_2} \tag{2.3}$$

$$L_3 = \sqrt{(x_n - u_{rc})^2 + (y_n - v_{rc})^2} - B\sqrt{Q_3} \tag{2.4}$$

where

$$x_n = \frac{r+q-p}{2\sqrt{r}} \tag{2.5}$$

$$y_n = \sqrt{q - x_n^2} \tag{2.6}$$

## 3. A New Six- to Four- Port Reduction Strategy

An overall reduction strategy that makes the best possible use of the new techniques is now described. A flow chart outlining the strategy is shown in figure 3.1.

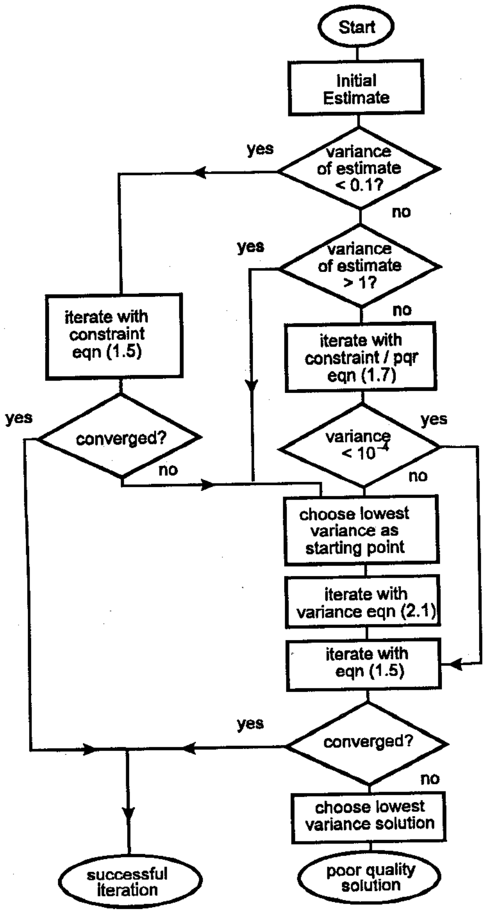


*Figure 3.1 : A new Six- to Four- Port Reduction strategy*

The best initial estimate is selected from closed - form calculations, theoretical values, and adjacent frequencies. The variance of these estimates is used to make the choice.

## 4. Accuracy of Solution

A computer simulation was used to evaluate the effect of noise on the six - to four - port reduction process. First, a simulated six- port was fully calibrated in the absence of noise. Next, the arbitrary termination data was re- taken with a controlled amount of random noise added. The reduction was then repeated for each of the methods using the same arbitrary termination data. Each reduction was tested by making a noise-free simulated measurement of a matched load (reflection coefficient = 0) using the noise- free four- port calibration. Ideally, an infinite return loss would be measured, so any inaccuracy in the reduction coefficients causes a finite residual measurement error.

Figure 4.1 shows how noise error in the arbitrary termination data propagates through the reduction coefficients to a residual error in reflection coefficient measurements. Reductions using (1.5) and (2.1) are both evaluated on an ideal and a poor six - port circuit, having circle centres at $1\angle 0°$, $1\angle 120°$, $1\angle -120°$, and $0.1\angle 0°$, $3\angle 108°$, $1\angle -161°$ respectively :-

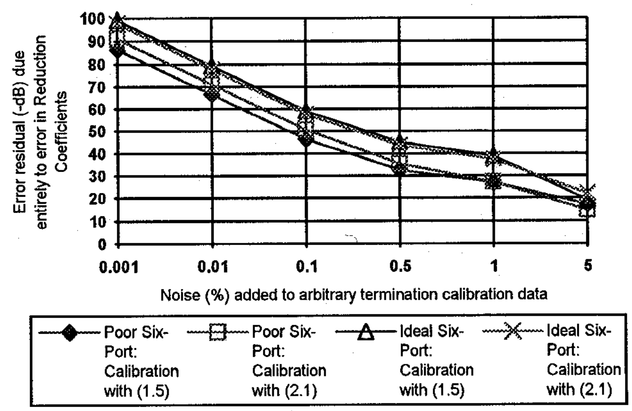


*Figure 4.1 : Error residual in reflection coefficient measurement, as a function of power measurement error in the arbitrary termination data used for the Six- to Four Port Reduction*

There is a slight improvement in quality of reduction coefficients when equation (2.1) is used. Equation (1.5) diverged from the true solution at the highest noise levels, and needed an artificially exact initial estimate to converge. Equation (2.1) did not fail in these tests.

Figure 4.2 shows the results of equation (2.1) after iteration is complete. This can be used to estimate the residual error in the reduction coefficients.

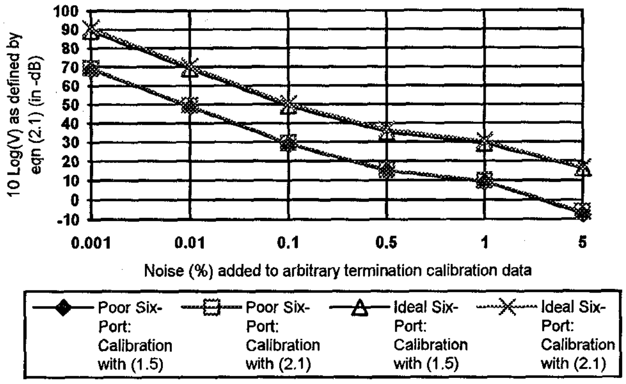


*Figure 4.2 : The use of Variance (equation 2.1) to estimate reflection coefficient measurement accuracy, considering only reduction coefficient errors*

## 5. Practical Results

The new six- to four- port reduction technique has been successfully implemented in a practical system[4]. The circuit is well- conditioned and the accuracy of the power ratios is < 0.1%. Most of the frequency points in the operating band of 250MHz - 26.5GHz converge using the constraint equation (1.5) in 3 to 5 iterations. At some frequency points, the variance iteration (2.1) is invoked, and the results checked by the constraint iteration (1.5). This occurs particularly at frequencies below 750MHz, which is outside the pass-band of the 3dB quadrature couplers employed. At all frequencies, a successful convergence is obtained. The final variance with the practical six- port system, shown in figure 5.1, is in the range $10^{-6}$ to $10^{-4}$ over the entire operating band.

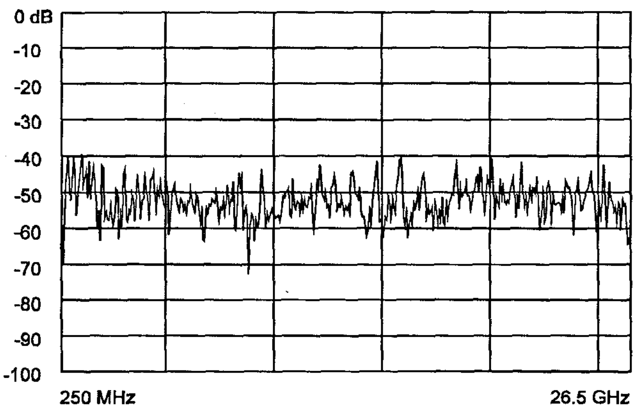


*Figure 5.1 : 10 Log (Variance) (equation 2.1) for a practical Six - Port Reflectometer*

Errors in the reduction coefficients contribute to the overall uncertainty in reflection coefficient measurement. The degree of contribution depends on the reflection coefficient of the device under test (DUT), and also on the four - port calibration method used, because errors only become apparent when the measurement is extrapolated away from the calibrated positions on the reflection coefficient plane. If a fixed or sliding load standard is used, the accuracy of the reduction coefficients has only a secondary effect on the effective directivity of the reflectometer. Conversely, the effective source match is directly influenced, and this is revealed particularly when measuring highly reflective devices whose phase lies between that of the calibration short circuit and open circuit. For the reflectometer described above, the effective source match and directivity are limited by the quality of the calibration pieces[4]:

| Connector Type | Source Match (dB) | Directivity (dB) |
|---|---|---|
| 3.5mm | 33 | 44 |
| 7mm | 42 | 53 |

*Table 5.1 : Effective source match and directivity for a practical Six - Port Reflectometer*

## Conclusions

A variance technique has been proposed that has two important applications. It can be used to quantify the accuracy of six- to four- port reduction coefficients, independently of the algorithm used to obtain them. In addition, it can form the basis of an enhanced reduction algorithm which is less susceptible to iteration divergence. This is important when noise on the power measurements or non-ideal circle centres cause the standard reduction iteration to diverge.

Full details of the new six- to four - port reduction strategy have been presented, together with practical results from a 250MHz to 26.5GHz co-axial six - port reflectometer.

## References


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